

On the geometric relativistic foundations of matter field theories and wave solutions as classic concepts

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PACS: 04.20.Cv · 04.20.Gz · 04.20.-q

Abstract

We consider geometric relativistic foundations, and we use them to define the basis upon which matter field theories are built; within this scheme we find wave solutions displaying the effects of the principle of exclusion: it is in terms of classic concepts that the effects of the principle of exclusion are understood. An order of magnitude of the scale is discussed.

Introduction

When we consider the geometrical relativistic foundations we see that they are established upon the existence of connections, which in the most general instance display torsion, although restrictions are imposed which induce the reduction to the completely antisymmetric part of torsion alone; considering completely antisymmetric torsion beside curvature for the spacetime sector, and gauge fields for the internal sector, we have the complete basis upon which the dynamics for matter fields can be built: for these matter field theories we can find special approximations, and within these approximations wave solutions can be obtained. Eventually these solutions will have properties that can be treated in terms of classic concepts.

In the present paper we will consider this approach for the spacetime-gauge matter field theories with wave solutions seen in the classic perspective, and we will push this description further than what is commonly done; in doing so we will see that these systems possess properties that can be described by taking into account the classic interpretation alone.

Finally we shall discuss these results in order to see how far this description can actually be pursued, or in what way problems may possibly arise.

1 Relativistic foundations of field theories and their wave solutions as classic concepts

In the geometry of relativistic coordinate tensors, coordinate tensors are defined by their transformation law under coordinate transformations, and a consequence of the fact that there are two transformations given as direct or inverse

then there are two possible different coordinate indices as upper and lower; upper indices are lowered by using the lowering procedure after the introduction of the coordinate tensor $g_{\alpha\beta}$ as well as lower indices are raised by using the raising procedure after the introduction of the coordinate tensor $g^{\alpha\beta}$, and these two tensors are symmetric and one the inverse of the other $g_{\nu\rho}g^{\rho\mu} = \delta_{\nu}^{\mu}$ thus they are called coordinate metric tensors. Dynamical properties for coordinate tensors are defined through coordinate covariant derivatives, so that after the introduction of the coordinate connections $\Gamma_{\mu\nu}^{\alpha}$ defined in term of their transformation law, the coordinate covariant derivative D_{μ} act on coordinate tensors yielding coordinate tensors; the most general coordinate connection is not symmetric in the two lower indices and it has a Cartan torsion tensor $Q_{\alpha\mu\rho}$ which will be considered to be completely antisymmetric $Q_{[\alpha\mu\rho]} = 6Q_{\alpha\mu\rho}$ in addition to the metricity condition $D_{\alpha}g = 0$ so that the coordinate connection will have one symmetric part written in terms of the coordinate metric tensor, with the consequence that the vanishing of the symmetric part of the coordinate connection and the flattening of the coordinate metric tensor coincide, thus entailing the principles of equivalence and causality, as discussed in [1].

From the coordinate metric tensor we can define the Levi-Civita completely antisymmetric tensor ε as usual; with this coordinate tensor we can write the completely antisymmetric torsion tensor in terms of the axial torsion vector as

$$Q_{\mu\rho\beta} = \varepsilon_{\mu\rho\beta\theta}W^{\theta} \quad (1)$$

and we also have $D_{\alpha}\varepsilon = 0$ identically.

Thus said we have that the expression

$$\Gamma_{\sigma\pi}^{\mu} = \frac{1}{2}g^{\mu\rho}[(\partial_{\pi}g_{\sigma\rho} + \partial_{\sigma}g_{\pi\rho} - \partial_{\rho}g_{\sigma\pi}) + Q_{\rho\sigma\pi}] \quad (2)$$

is a decomposition showing that the coordinate connections $\Gamma_{\mu\nu}^{\alpha}$ can actually be separated into a coordinate connection $\Lambda_{\mu\nu}^{\alpha}$ plus torsion tensors, and so the coordinate covariant derivatives D_{μ} will be separated into the coordinate covariant derivative ∇_{μ} acting on coordinate tensors to yield coordinate tensors plus torsional contributions; the coordinate connection $\Lambda_{\mu\nu}^{\alpha}$ will then be symmetric in the two lower indices thus torsionless and again verifying the metricity condition by construction as it can be directly checked.

Considering the coordinate connection it is possible to define

$$G_{\rho\sigma\pi}^{\mu} = \partial_{\sigma}\Gamma_{\rho\pi}^{\mu} - \partial_{\pi}\Gamma_{\rho\sigma}^{\mu} + \Gamma_{\lambda\sigma}^{\mu}\Gamma_{\rho\pi}^{\lambda} - \Gamma_{\lambda\pi}^{\mu}\Gamma_{\rho\sigma}^{\lambda} \quad (3)$$

which is a coordinate tensor antisymmetric in the first and second couple of indices called Riemann tensor.

Because of these symmetry properties Riemann tensor has one independent contraction conventionally given by $G_{\rho\alpha\sigma}^{\alpha} = G_{\rho\sigma}$ which has one contraction given by $G_{\rho\sigma}g^{\rho\sigma} = G$ called Ricci tensor and scalar respectively.

And because of the decomposition above we can write

$$G_{\rho\sigma\pi}^{\mu} = R_{\rho\sigma\pi}^{\mu} + \frac{1}{2}(\nabla_{\sigma}Q_{\rho\pi}^{\mu} - \nabla_{\pi}Q_{\rho\sigma}^{\mu}) + \frac{1}{4}(Q_{\lambda\sigma}^{\mu}Q_{\rho\pi}^{\lambda} - Q_{\lambda\pi}^{\mu}Q_{\rho\sigma}^{\lambda}) \quad (4)$$

where the torsionless curvature tensor $R_{\rho\alpha\sigma}^{\alpha}$ is separated from torsion tensor.

With this expression of Cartan torsion tensor we have that the expression for the Riemann tensors has been defined in this way in order to have the

commutator of two coordinate covariant derivatives to be given by the following

$$\begin{aligned}
[D_\zeta, D_\theta]T_{\beta\dots\rho}^{\alpha\dots\sigma} &= Q^\mu_{\zeta\theta}D_\mu T_{\beta\dots\rho}^{\alpha\dots\sigma} + \\
&+ \left(T_{\beta\dots\rho}^{\nu\dots\sigma}G^\alpha_{\nu\zeta\theta} + \dots + T_{\beta\dots\rho}^{\alpha\dots\nu}G^\sigma_{\nu\zeta\theta} \right) - \\
&- \left(T_{\nu\dots\rho}^{\alpha\dots\sigma}G^\nu_{\beta\zeta\theta} + \dots + T_{\beta\dots\nu}^{\alpha\dots\sigma}G^\nu_{\rho\zeta\theta} \right)
\end{aligned} \tag{5}$$

which is a geometric identity.

The commutator of three coordinate covariant derivatives in cyclic permutations also gives the expression

$$\begin{aligned}
&(D_\kappa Q^\rho_{\mu\nu} + Q^\rho_{\kappa\pi}Q^\pi_{\mu\nu} + G^\rho_{\kappa\mu\nu}) + (D_\nu Q^\rho_{\kappa\mu} + Q^\rho_{\nu\pi}Q^\pi_{\kappa\mu} + G^\rho_{\nu\kappa\mu}) + \\
&+ (D_\mu Q^\rho_{\nu\kappa} + Q^\rho_{\mu\pi}Q^\pi_{\nu\kappa} + G^\rho_{\mu\nu\kappa}) \equiv 0
\end{aligned} \tag{6}$$

called torsion Jacobi-Bianchi identities and

$$\begin{aligned}
&(D_\mu G^\nu_{\iota\kappa\rho} - G^\nu_{\iota\beta\mu}Q^\beta_{\kappa\rho}) + (D_\kappa G^\nu_{\iota\rho\mu} - G^\nu_{\iota\beta\kappa}Q^\beta_{\rho\mu}) + \\
&+ (D_\rho G^\nu_{\iota\mu\kappa} - G^\nu_{\iota\beta\rho}Q^\beta_{\mu\kappa}) \equiv 0
\end{aligned} \tag{7}$$

called curvature Jacobi-Bianchi identities.

These identities are such that the former has one contraction

$$D_\rho Q^{\rho\mu\nu} + (G^{\nu\mu} - \frac{1}{2}g^{\nu\mu}G) - (G^{\mu\nu} - \frac{1}{2}g^{\mu\nu}G) \equiv 0 \tag{8}$$

called fully contracted torsion Jacobi-Bianchi identities while the latter has one independent contraction

$$D_\mu G^\mu_{\iota\kappa\rho} - D_\kappa G_{\iota\rho} + D_\rho G_{\iota\kappa} + G_{\iota\beta}Q^\beta_{\kappa\rho} - G^\mu_{\iota\beta\kappa}Q^\beta_{\rho\mu} + G^\mu_{\iota\beta\rho}Q^\beta_{\kappa\mu} \equiv 0 \tag{9}$$

called contracted curvature Jacobi-Bianchi identities with contraction

$$D_\rho (G^{\rho\kappa} - \frac{1}{2}g^{\rho\kappa}G) + (G_{\rho\beta} - \frac{1}{2}g_{\rho\beta}G) Q^{\rho\beta\kappa} + \frac{1}{2}Q_{\nu\rho\beta}G^{\nu\rho\beta\kappa} \equiv 0 \tag{10}$$

known as fully contracted curvature Jacobi-Bianchi identities.

Finally we have that

$$\nabla_\mu R^\nu_{\iota\kappa\rho} + \nabla_\kappa R^\nu_{\iota\rho\mu} + \nabla_\rho R^\nu_{\iota\mu\kappa} \equiv 0 \tag{11}$$

are known as the torsionless curvature Jacobi-Bianchi identities.

Given the coordinate metric tensors $g_{\alpha\beta}$ and $g^{\alpha\beta}$ it is possible to define covariant metric concepts like lengths and angles, and so considered a pair of bases of vectors called vierbeins ξ^a_β and ξ^β_a defined to be dual of one another $\xi^a_\mu \xi^\mu_a = \delta^\rho_\rho$ and $\xi^\mu_a \xi^\mu_r = \delta^a_r$ we can always choose them to be such that they verify the orthonormality conditions given by $\xi^a_\sigma \xi^b_\rho g^{\sigma\rho} = \eta^{ab}$ equivalently given by $\xi^\sigma_a \xi^\rho_b g_{\sigma\rho} = \eta_{ab}$ where η_{ab} and η^{ab} are unitary diagonal matrices called Minkowskian matrices: although it is always possible to orthonormalize a basis of vectors so that it is without loss of generality that orthonormal vierbeins are introduced, nevertheless they are determined up to a Lorentz transformation that can be made explicit; the introduction of the vierbeins is essential because after multiplying a coordinate tensor by the vierbein and contracting their coordinate indices we are left with a world index in a world tensor so

that the transformation law for coordinate tensors in terms of the most general coordinates transformation becomes a transformation law for world tensors in terms of a Lorentz transformation of explicit form. So within the geometry of relativistic world tensors, the world tensors are defined in terms of their transformation law under Lorentz transformations, and also in this case the two possible Lorentz transformations give two possible different world indices; the lowering and raising world indices procedure is done by means of η_{ab} and η^{ab} , which are symmetric and they are reciprocal of one another $\eta_{ab}\eta^{bm} = \delta_a^m$ and therefore they are known as Minkowskian metric matrices. And dynamical properties for world tensors are defined through world covariant derivatives, and thus after the introduction of the Lorentz connection given by $\Gamma_{j\mu}^i$ and defined by its transformation law, the world covariant derivative D_μ acts on world tensors giving world tensors; the condition $D_\alpha \xi = 0$ is imposed to make the coordinate and world covariant derivatives coincide, and the vanishing of the covariant derivative of the Minkowskian metric matrices $D_\alpha \eta = 0$ is given automatically.

These two conditions in fact imply that the Lorentz connection is written as

$$\Gamma_{j\mu}^b = \xi_j^\alpha \xi_\rho^b (\Gamma_{\alpha\mu}^\rho + \xi_\alpha^k \partial_\mu \xi_k^\rho) \quad (12)$$

in terms of the coordinate connection and antisymmetric in the world indices.

Considering the Lorentz connection it is possible to define

$$G_{b\sigma\pi}^a = \partial_\sigma \Gamma_{b\pi}^a - \partial_\pi \Gamma_{b\sigma}^a + \Gamma_{j\sigma}^a \Gamma_{b\pi}^j - \Gamma_{j\pi}^a \Gamma_{b\sigma}^j \quad (13)$$

which is a tensor that is antisymmetric in both the coordinate and the world indices and it is related to the Riemann tensor as we shall see.

Indeed we have that this tensor is writable as

$$G_{b\sigma\pi}^a = G_{\rho\sigma\pi}^\mu \xi_b^\rho \xi_\mu^a \quad (14)$$

in terms of the Riemann tensor as it should have been obvious.

With this expression for the Riemann tensor the commutator of two covariant derivatives is

$$\begin{aligned} [D_\zeta, D_\theta] T_{\beta\dots\rho b\dots r}^{\alpha\dots\sigma a\dots s} &= Q_{\zeta\theta}^\mu D_\mu T_{\beta\dots\rho b\dots r}^{\alpha\dots\sigma a\dots s} + \\ &+ \left(T_{\beta\dots\rho b\dots r}^{\nu\dots\sigma a\dots s} G_{\nu\zeta\theta}^\alpha + \dots + T_{\beta\dots\rho b\dots r}^{\alpha\dots\nu a\dots s} G_{\nu\zeta\theta}^\sigma \right) - \\ &- \left(T_{\nu\dots\rho b\dots r}^{\alpha\dots\sigma a\dots s} G_{\beta\zeta\theta}^\nu + \dots + T_{\beta\dots\nu b\dots r}^{\alpha\dots\sigma a\dots s} G_{\rho\zeta\theta}^\nu \right) + \\ &+ \left(T_{\beta\dots\rho b\dots r}^{\alpha\dots\sigma j\dots s} G_{j\zeta\theta}^a + \dots + T_{\beta\dots\rho b\dots r}^{\alpha\dots\sigma a\dots j} G_{j\zeta\theta}^s \right) - \\ &- \left(T_{\beta\dots\rho j\dots r}^{\alpha\dots\sigma a\dots s} G_{b\zeta\theta}^j + \dots + T_{\beta\dots\rho b\dots j}^{\alpha\dots\sigma a\dots s} G_{r\zeta\theta}^j \right) \end{aligned} \quad (15)$$

which is again a geometric identity.

The introduction of the vierbein and the Lorentz tensors is important because whereas nothing can be said about the most general coordinate transformation nevertheless Lorentz transformations have an explicit structure, and consequently the same structure can also be given obtained in terms of other representations; of all these representations, the most special are the complex valued representations: these complex representations of the Lorentz transformations are called spinorial transformations \mathbf{S} and we have that they can be

expanded in terms of their infinitesimal generators given by σ_{ij} according to the following expression

$$\sigma_{ij} = \frac{1}{4}[\gamma_i, \gamma_j] \quad (16)$$

in terms of a set of γ_a matrices such that

$$\{\gamma_i, \gamma_j\} = 2\mathbb{I}\eta_{ij} \quad (17)$$

where we have that \mathbb{I} is the identity matrix and $\gamma = i\gamma^0\gamma^1\gamma^2\gamma^3$ is the dual of the identity matrix itself; once the spinorial transformation \mathbf{S} is given, we can define the complex fields that transform according to this transformation as spinor fields, classified in terms of half-integer spin for which we will restrict ourselves to the case given by the $\frac{1}{2}$ -spin spinor fields alone. And so within this geometry of relativistic spinor fields, spinor fields are defined as transforming according to the transformation law given by $\psi' = \mathbf{S}\psi$ or $\bar{\psi}' = \bar{\psi}\mathbf{S}^{-1}$ for the inverse; the passage between the two forms is defined by the adjoint procedure in terms of the γ_0 matrix as $\bar{\psi} \equiv \psi^\dagger\gamma_0$ or $\psi \equiv \gamma_0\bar{\psi}^\dagger$ reciprocally. Dynamical properties for spinor fields are defined through covariant derivatives, so that after the introduction of the spinorial connection Γ_μ defined in terms of its transformation law, the spinorial covariant derivative \mathbf{D}_μ acts on spinor fields yielding spinor fields; then the γ_j and σ_{ij} matrices are constant automatically.

Then the spinorial connection is written as

$$\Gamma_\mu = \frac{1}{2}\Gamma_{\mu}^{ab}\sigma_{ab} \quad (18)$$

in terms of the Lorentz connection and with no symmetries in its indices because no transposition of its indices can be defined.

Considering the spinorial connection it is instead still possible to define

$$\mathbf{G}_{\sigma\pi} = \partial_\sigma\Gamma_\pi - \partial_\pi\Gamma_\sigma + [\Gamma_\sigma, \Gamma_\pi] \quad (19)$$

which is a tensorial spinor antisymmetric in the tensorial indices and which we may call Riemann tensorial spinor.

In fact this tensorial spinor is

$$\mathbf{G}_{\sigma\pi} = \frac{1}{2}G_{\sigma\pi}^{ab}\sigma_{ab} \quad (20)$$

in terms of the Riemann tensor identically.

With this expression for the Riemann tensorial spinor the commutator of two covariant derivatives is given by

$$[\mathbf{D}_\zeta, \mathbf{D}_\theta]\psi = Q^\mu_{\zeta\theta}\mathbf{D}_\mu\psi + \mathbf{G}_{\zeta\theta}\psi \quad (21)$$

which is a geometric identity for the spinor field.

It is important to notice that because we have complex valued fields defined up to a complex phase then the problem of their gauge invariance has to be considered as well; under this point of view then, the gauge invariance is introduced as a geometrical invariance.

So within this geometry of relativistic complex fields, complex fields are defined according to the transformation law $\psi' = e^{iq\theta}\psi$ or $\bar{\psi}' = e^{-iq\theta}\bar{\psi}$ where we have that q is defined to be the charge label. Dynamical properties for complex

fields are defined through gauge derivatives, so that after the introduction of the phase connection $\mathbf{\Gamma}_\mu$ defined in terms of its transformation law, the gauge derivative \mathbf{D}_μ acts on complex fields yielding complex fields.

Considering the phase connection it is possible to define

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (22)$$

which is a tensor antisymmetric in the two indices called Maxwell tensor.

With the Maxwell tensor the commutator of two gauge derivatives is

$$[D_\zeta, D_\theta]\psi = iqF_{\zeta\theta}\psi \quad (23)$$

which is a geometric identity for the spinor field.

The commutator of three gauge derivatives in cyclic permutations gives

$$\partial_\mu F_{\kappa\rho} + \partial_\kappa F_{\rho\mu} + \partial_\rho F_{\mu\kappa} \equiv 0 \quad (24)$$

known as Jacobi-Cauchy identities.

And a special case of the commutator is the commutator of the Maxwell tensor after its full contraction given by

$$D_\rho (D_\sigma F^{\sigma\rho} + \frac{1}{2}Q^{\rho\alpha\mu}F_{\alpha\mu}) = 0 \quad (25)$$

and this is an identity as well.

Now we notice that the spinorial connection we have written was not the most general spinorial connection because it could actually be generalized by the addition of a term proportional to the identity matrix just like the one provided by the phase connection, so that the spinorial connection given by

$$\mathbf{A}_\mu = iqA_\mu\mathbb{I} + \mathbf{\Gamma}_\mu \quad (26)$$

combines the spinorial connection with the phase connection in what is the connection in its most general form possible.

Considering this connection it is possible to define

$$\mathbf{F}_{\sigma\pi} = \partial_\sigma \mathbf{A}_\pi - \partial_\pi \mathbf{A}_\sigma + [\mathbf{A}_\sigma, \mathbf{A}_\pi] \quad (27)$$

which is a tensorial spinor antisymmetric in the tensorial indices.

And this tensorial spinor is writable as

$$\mathbf{F}_{\sigma\pi} = iqF_{\sigma\pi}\mathbb{I} + \mathbf{G}_{\sigma\pi} \quad (28)$$

as a combination of both the Riemann tensorial spinor and the Maxwell tensor expressed in what is a considerably compact form.

With this compact expression then the commutator of two gauge covariant derivatives is given by

$$[\mathbf{D}_\zeta, \mathbf{D}_\theta]\psi = Q^\mu_{\zeta\theta}\mathbf{D}_\mu\psi + \mathbf{F}_{\zeta\theta}\psi \quad (29)$$

and this is a geometric identity for the spinor field we will use in the following.

In this way we have that the Jacobi-Bianchi identities in their fully contracted form for the torsion (8) and for the curvature (10) and also the identity of the commutator of the Maxwell tensor in its fully contracted form (25) are

satisfied through the identity of the commutator of the spinor field (29) if we postulate the system of field equations given by the torsion coupling to the spin

$$Q^{\rho\mu\nu} = -\frac{i}{4}\bar{\psi}\{\gamma^\rho, \sigma^{\mu\nu}\}\psi \quad (30)$$

and the curvature coupling to the energy

$$G^\mu{}_\nu - \frac{1}{2}\delta^\mu_\nu G - \frac{1}{8}\delta^\mu_\nu F^2 + \frac{1}{2}F^{\rho\mu}F_{\rho\nu} = \lambda\delta^\mu_\nu + \frac{i}{4}(\bar{\psi}\gamma^\mu D_\nu\psi - D_\nu\bar{\psi}\gamma^\mu\psi) \quad (31)$$

and also the gauge field coupling to the current

$$D_\sigma F^{\sigma\rho} + \frac{1}{2}F_{\mu\nu}Q^{\mu\nu\rho} = q(\bar{\psi}\gamma^\rho\psi) \quad (32)$$

together with the matter field equation

$$i\gamma^\mu D_\mu\psi - (m\mathbb{I} + ib\gamma)\psi = 0 \quad (33)$$

in terms of the system of integration constants λ , m and b , as discussed in [2].

In this system of field equations it is possible to separate torsion and because the field equation for the torsion coupling to the spin is algebraic then we can substitute torsion with the spin written in terms of the matter field to get

$$\begin{aligned} R_{\mu\nu} = & \frac{1}{8}g_{\mu\nu}F^2 - \frac{1}{2}g^{\eta\rho}F_{\mu\eta}F_{\nu\rho} - \lambda g_{\mu\nu} - \frac{1}{4}[m(\bar{\psi}\psi) + b(i\bar{\psi}\gamma\psi)]g_{\mu\nu} + \\ & + \frac{i}{8}(\bar{\psi}\gamma_\mu\nabla_\nu\psi + \bar{\psi}\gamma_\nu\nabla_\mu\psi - \nabla_\nu\bar{\psi}\gamma_\mu\psi - \nabla_\mu\bar{\psi}\gamma_\nu\psi) \end{aligned} \quad (34)$$

and

$$\nabla_\sigma F^{\sigma\rho} = q(\bar{\psi}\gamma^\rho\psi) \quad (35)$$

along with

$$\begin{aligned} i\gamma^\mu\nabla_\mu\psi - \frac{3}{16}[(\bar{\psi}\psi)\mathbb{I} + i(i\bar{\psi}\gamma\psi)\gamma]\psi - (m\mathbb{I} + ib\gamma)\psi \equiv \\ \equiv i\gamma^\mu\nabla_\mu\psi - \frac{3}{16}(\bar{\psi}\gamma_\mu\psi)\gamma^\mu\psi - (m\mathbb{I} + ib\gamma)\psi = 0 \end{aligned} \quad (36)$$

where the field equations for the curvature coupling to the energy are equal to the field equations for the curvature coupling to the energy in the torsionless case and the field equations for the gauge field coupling to the current are equal to the field equations for the gauge field coupling to the current in the torsionless case but the field equations for the matter field are equivalent to the equations for the matter field in the torsionless case with additional potentials that are given by specific fields constructed in terms of the matter fields themselves.

In the following we shall study this system of field equations; and among all field equations we shall focus our attention on the matter field equation so to study its properties: eventually this equation will be solved to get some of its special solutions in the form of plane waves so to investigate which properties they display and to see what consequences these properties have.

1.1 Theories and their solutions in classic perspective

Because of the principle of equivalence and causality there exists one system of coordinates in which locally the metric is constant and the connection is vanishing so that the whole connection is negligible and the covariant derivative

is the partial derivative calculated with respect to the spacetime position that is expressed as in the ordinary circumstance of macroscopic situations.

We notice that the field equations for the curvature coupling to the energy may be written as

$$R_{\mu\nu} = \frac{1}{8}g_{\mu\nu}F^2 - \frac{1}{2}g^{\eta\rho}F_{\mu\eta}F_{\nu\rho} - \left[\lambda - \frac{3}{64}(\bar{\psi}\psi)^2\right]g_{\mu\nu} - \frac{1}{4}\left[m + \frac{3}{16}(\bar{\psi}\psi)\right](\bar{\psi}\psi)g_{\mu\nu} + \frac{1}{2}\left[m + \frac{3}{16}(\bar{\psi}\psi)\right](\bar{\psi}\psi)U_\mu U_\nu \quad (37)$$

where all potentials of matter fields are written as shifts for the integration constants $64\Lambda = 64\lambda - 3(\bar{\psi}\psi)^2$ and $16M = 16m + 3(\bar{\psi}\psi)$ like modified parameters for which once the mass density is $\mu = M(\bar{\psi}\psi)$ then we obtain the field equations for the curvature coupling to the energy density in the macroscopic situation.

For the field equations of the gauge field coupling to the current we have

$$\nabla_\sigma F^{\sigma\rho} = \pm q(\bar{\psi}\psi)U^\rho \quad (38)$$

for which once the charge density is $\rho = q(\bar{\psi}\psi)$ we obtain the field equations for the gauge field coupling to the current density in the macroscopic situation.

Finally for the matter field equations we have that

$$\begin{aligned} & i\gamma^\mu \nabla_\mu \psi - \frac{3}{16}(\bar{\psi}\psi)\psi - m\psi \equiv \\ & \equiv i\gamma^\mu \nabla_\mu \psi - \frac{3}{16}(\bar{\psi}\gamma_\mu \psi)\gamma^\mu \psi - m\psi = 0 \end{aligned} \quad (39)$$

and from which it is clear that this matter field equation is the Dirac matter field equation in Nambu-Jona-Lasinio or in Gross-Neveu or Thirring form, as it has been discussed by these authors respectively in [3] and [4] or in [5] or [6].

When in this matter field equation we separate the spatial and the temporal components it is possible to take the stationary configuration and the limit in which the velocity tends to zero so that the small semispinorial component vanishes and the large semispinorial component ϕ is subject to the approximated matter field equation

$$\begin{aligned} & i\frac{\partial\phi}{\partial t} + \frac{1}{2m}\nabla^a\nabla_a\phi - \frac{3}{512m}(\phi^\dagger\phi)(\phi\phi^\dagger)\phi \mp \frac{1}{4}(\phi^\dagger\phi)\phi \equiv \\ & \equiv i\frac{\partial\phi}{\partial t} + \frac{1}{2m}\nabla^a\nabla_a\phi - \frac{3}{512m}(\phi^\dagger\sigma^a\phi)(\phi\phi^\dagger)\sigma_a\phi \mp \frac{1}{4}(\phi^\dagger\sigma^a\phi)\sigma_a\phi = 0 \end{aligned} \quad (40)$$

and this approximated matter field equation is the Schroedinger matter field equation in the Heisenberg form, as it has been discussed by this author in [7].

We remark that in the matter field equation the non-linear term has the scalar form of the mass term or the vectorial form of the electrodynamic potential and consequently describing a matter field generating its mass term or its electrodynamic potential, so that the dynamics is qualitatively that of a matter field with the mass term in the dispersion relations or interacting with its electrodynamic field in a diffusing back-reaction; moreover in the approximated matter field equation the non-linear term has the features of a rotational potential, so that the dynamics is qualitatively that of a matter field scattering against a centrifugal barrier arising from its self-interaction: all this suggests that in the matter field equation the non-linear terms are such that the dynamics is qualitatively that of a matter field spreading due to its repulsive autointeractions.

Now we observe that in order for the field equations for the curvature coupling to the energy to be given by equation (37) and for the gauge field coupling

to the current to be given by equation (38) it is necessary that the matter field equation given by equation (39) is solved in terms of the plane wave solutions written in either of the two expressions given by the following

$$\psi = e^{-ix^\mu P_\mu \left(1 + \frac{3A^2}{16m}\right)} \begin{bmatrix} A \cos \frac{\theta}{2} \left(\sqrt{\frac{E+P}{4m}} + \sqrt{\frac{E-P}{4m}} \right) \\ A \sin \frac{\theta}{2} \left(\sqrt{\frac{E-P}{4m}} + \sqrt{\frac{E+P}{4m}} \right) \\ A \cos \frac{\theta}{2} \left(\sqrt{\frac{E+P}{4m}} - \sqrt{\frac{E-P}{4m}} \right) \\ A \sin \frac{\theta}{2} \left(\sqrt{\frac{E-P}{4m}} - \sqrt{\frac{E+P}{4m}} \right) \end{bmatrix} \quad (41)$$

$$\psi = e^{ix^\mu P_\mu \left(1 - \frac{3A^2}{16m}\right)} \begin{bmatrix} A \cos \frac{\theta}{2} \left(\sqrt{\frac{E+P}{4m}} - \sqrt{\frac{E-P}{4m}} \right) \\ A \sin \frac{\theta}{2} \left(\sqrt{\frac{E-P}{4m}} - \sqrt{\frac{E+P}{4m}} \right) \\ A \cos \frac{\theta}{2} \left(\sqrt{\frac{E+P}{4m}} + \sqrt{\frac{E-P}{4m}} \right) \\ A \sin \frac{\theta}{2} \left(\sqrt{\frac{E-P}{4m}} + \sqrt{\frac{E+P}{4m}} \right) \end{bmatrix} \quad (42)$$

having x^μ as spacetime position with A a scale factor and θ the angle between the spin and the momentum conventionally taken to be along the third axis and where there are corrections in the frequencies due to spinorial autointeractions.

In the same way the matter field equation given by (40) is solved in terms of the plane wave solutions written as in the following

$$\phi = e^{-i \left[t \left(\frac{P^2}{2m} + \frac{3A^4}{512m} \pm \frac{A^2}{4} \right) - x^a P_a \right]} \begin{bmatrix} A \cos \frac{\theta}{2} \\ A \sin \frac{\theta}{2} \end{bmatrix} \quad (43)$$

having t as time and x^a as space position with A a scale factor and θ the angle between the directions of the spin and the momentum conventionally taken to be along the third axis and in which we see that there are corrections in the frequency associated to the energy due to the effect of spinorial autointeractions.

Finally we take into account the solutions (41) and (42) and we notice that for each of these two solutions there are two components which are independent as they correspond to the two opposite eigenvalues of the third component of the spin operator; also for the approximated solution (43) the two components are independent corresponding to the two opposite eigenvalues of the third component of the spin operator: hence it is clear that for two solutions with opposite third component of the spin there is a special linear combination that is still a solution; on the other hand however we know that solutions in linear combination are not solutions for non-linear equations in general: henceforth two solutions with opposite third component of the spin may superpose although superposition is impossible in general, thus entailing the repulsive effects of the principle of exclusion, as it has also been alternatively discussed by Sachs in [8].

In the following section we shall focus our attention on this set of matter field solutions: these solutions will be discussed in terms of the classic perspective.

1.1.1 Solutions seen by means of the classic interpretation

Because we discussed special cases their peculiar properties cannot be extend to more general systems, although it is interesting to speculate about what would happen in the nevertheless reasonable assumption that these were actually properties of general systems: the first main result we have found is that we have

obtained the matter field equation given by the Dirac matter field equation as in Nambu-Jona-Lasinio or in Gross-Neveu or Thirring form, with an approximation for which we got the matter field equation as the Schroedinger matter field equation in the Heisenberg form, both having spinorial potentials giving rise to repulsive dynamics; of these matter field equations we have found plane wave matter field solutions, whose specific form was able to give rise to the repulsive effects of the principle of exclusion. In this way, we have that it is in terms of constraints upon matter fields that the repulsive effects of the principle of exclusion are comprehended within the scheme of the classic interpretation.

Also, we remark that having gravitation, electrodynamics and matter fields requires three fundamental constants to set their strengths: in fact by fixing the gravitational constant to the value $8\pi G = 1$ we adjust the strength of the gravitation compared to all other fields, the value of the charge adjusts the strength of electrodynamics compared to the remaining field, and then the Planck constant normalized to unity assigns to the matter field its normalization in an absolute way; the fact that these fields have different relative strengths and absolute normalizations is a consequence of the fact that they are independent fields. On the other hand the fundamental constant that gives to the matter field its normalization may be written in terms of the scale factor; accordingly by assuming the scale factor to be of the order of magnitude of $A^2 = 10^{-22}$ in units of Planck then it is at scales of the order of magnitude of 10^{22} lengths of Planck that the effects we have discussed are present, and gravitation might become comparable to electrodynamics acting upon the matter fields at the microscopic level, entailing a running of the coupling constants analogous to that described by the Dirac hypothesis, as it has been discussed by Dirac in [9].

What this analysis has shown is that the matter field solutions we have found behave in such a way that is constrained to produce the effects of the principle of exclusion with the classic interpretation and that a scale factor may be chosen in order for these gravitational effects to be manifest at microscopic level; this suggests the possibility for which these gravitational forces for the spinor field may be able to reproduce the same effects imposed by the anticommutation relations between the components of the spinor field itself. In this way it would be possible to employ gravitational physics in order to reproduce the properties that are usually ascribed by quantum physics, and consequently it would be possible to speculate that gravitation is a classical field that appears to affect the dynamics of fields in the same way in which quantization would do. Admittedly this speculation may well be totally wrong but nevertheless it seems reasonable and certainly it is intriguing and too important to be left without a thorough study, although on the other hand it is our opinion that such discussion would be beyond the scope of the present article.

Conclusion

In this paper we have been employing only geometric relativistic identities to get the system of field equations in which the Dirac matter field equation had the Nambu-Jona-Lasinio or the Gross-Neveu or Thirring form together with its approximation which was given by the Schroedinger matter field equation having the Heisenberg form, and we have obtained the corresponding plane wave matter field solutions with interactions of repulsive type; these matter

fields behave according to the effects of the principle of exclusion: eventually a rough estimate has been performed in order to show that this dynamical behavior may already be manifest at scales of the nucleus thus strengthening the idea of a description given in terms of the classic interpretation.

In the end we have found that it appears that the most general dynamical behavior of the classical theory is analogous to what is attributed by quantum models; however so far the extension of this analogy to reproduce the complete phenomenology is a conjecture and more information can be obtained only through a deeper study. A following paper will be written.

References

- [1] L. Fabbri, in *Contemporary Fundamental Physics* (Nova Science Publishers, New York, 2010).
- [2] L. Fabbri, in *Contemporary Fundamental Physics* (Nova Science Publishers, New York, 2011).
- [3] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961).
- [4] Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **124**, 246 (1961).
- [5] D. J. Gross and A. Neveu, *Phys. Rev.* **D10**, 3235 (1974).
- [6] W. E. Thirring, *Annals Phys.* **3**, 91 (1958).
- [7] W. Heisenberg, *Z. Naturforschung* **14**, 441 (1959).
- [8] M. Sachs, *Nuovo Cim. B* **44**, 289 (1978).
- [9] P. A. M. Dirac, *Proc. Roy. Soc. A* **338**, 439 (1974).